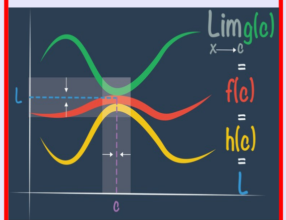
Math 261

Fall 2022

Lecture 24



Math 261
Class Quiz 10

Name: ____________________

No Work ⇔ No Points

Use Pencil Only ⇔ Be Neat & Organized

1. (4 points) Find $f'(x)$ **for** $f(x) = 2\cos^2 x^3 \tan x^3$.

$f(x) = 2\cos x^3 \cdot \cos x^3 \cdot \operatorname{Tan} x^3$

$= 2\cos x^3 \cdot \sin x^3 = \sin 2x^3$

$f'(x) = \cos 2x^3 \cdot 6x^2$

$\boxed{f'(x) = 6x^2 \cos 2x^3}$

1. ______

2. (5 points) Find $f'(x)$ **for** $f(x) = \frac{1}{2}(x^2-1)^{-2}$.

$f'(x) = \frac{1}{\cancel{2}} \cdot (-\cancel{2})(x^2-1)^{-3} \cdot 2x$

$= \frac{-2x}{(x^2-1)^3}$

$f'(x) = \frac{-2x}{(x^2-1)^3}$

2. ______

3. (4 points) Find $\frac{dy}{dx}$ **for** $y^3 = \sqrt{xy}$.

$y^6 = xy$

If $y \neq 0$

$y^5 = x$

$5y^4 \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{5y^4}$

If $y = 0$

$\frac{dy}{dx} = 0$

3. ______

4. (7 points) Find $\frac{dx}{dy}$ **for** $x^2y - xy^2 = 10$.

$2x \cdot \frac{dx}{dy} \cdot y + x^2 \cdot 1 - \left(\frac{dx}{dy} y^2 + x \cdot 2y\right) = 0$

$2xy \frac{dx}{dy} - y^2 \frac{dx}{dy} = 2xy - x^2$

$\boxed{\frac{dx}{dy} = \frac{2xy - x^2}{2xy - y^2}}$

4. ______

Page 1 of 1 Class Quiz 10 Total Points: 20

Related Rates:

Let's say we have $x^2 + y^2 = z^2$ and $x, y,$ and z all change with respect to time.

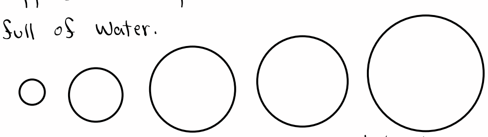
This means $x = x(t)$, $y = y(t)$, and $z = z(t)$.

$$\frac{d}{dt}\left[x^2 + y^2 = z^2\right]$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$$

Solve $\dfrac{dz}{dt} = \dfrac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$

Suppose we drop a stone in a bathtub full of water.



wave created by dropping that stone creates circular motion.

Area of such circles are given by

$$A = \pi r^2$$

As time increases, Radius increases as well as the area of these circles.

So $r = r(t)$, $A = A(t)$

$$\frac{d}{dt}\left[A = \pi r^2\right] \qquad \frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Suppose r changes at 2cm/min. $\frac{dr}{dt} = 2$

How fast is the area changing when $r = 5$cm. $\to \frac{dA}{dt} = ?$

$$\frac{dA}{dt} = \underset{2\pi}{2\pi}\,\underset{r}{(5)} \cdot \underset{\frac{dr}{dt}}{2} = \boxed{20\pi \text{ cm}^2/\text{min.}}$$

A basketball is losing air at the rate of 5 cm^3/min.

$\frac{dV}{dt} = -5 cm^3/min.$

How fast is its radius changing when the radius is 2 cm?

$\frac{dr}{dt} = ?$

$\boxed{r = 2 cm.}$

Volume of Sphere

$V = \frac{4\pi r^3}{3}$

$V = V(t)$

$r = r(t)$

$\frac{d}{dt}\left[V = \frac{4\pi}{3} r^3\right]$

$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3} r^2 \cdot \frac{dr}{dt}$

$-5 = 4\pi(2)^2 \cdot \frac{dr}{dt}$

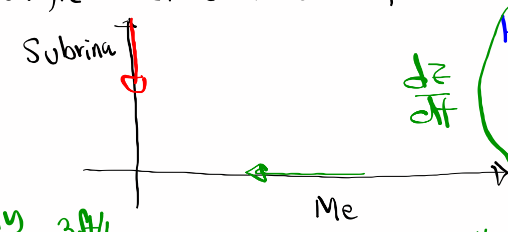
$\frac{dr}{dt} = \frac{-5}{16\pi}$ cm/min

r is decreasing.

I walk 4 ft/min.

Sabrina walks 3 ft/min.

we are getting closer to each other, but angle between our paths is 90°.



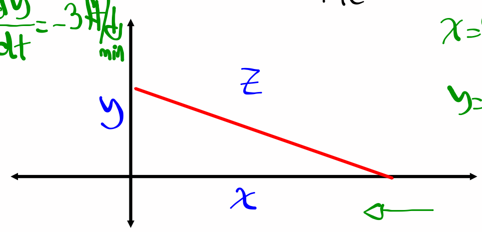
$\frac{dz}{dt}$

How fast is the distance between us changing when I am 4 ft & Sabrina is 3 ft from the intersection?

$x = 4$

$y = 3$

$\frac{dy}{dt} = -3$ ft/min



$\boxed{x^2 + y^2 = z^2}$

$\frac{dx}{dt} = -4$ ft/min.

$$\frac{d}{dt}\left[x^2+y^2=z^2\right]$$

$$\cancel{2}x\cdot\boxed{\frac{dx}{dt}}+\cancel{2}y\cdot\boxed{\frac{dy}{dt}}=\cancel{2}z\cdot\frac{dz}{dt}$$

$x^2+y^2=z^2$

$4^2+3^2=z^2$

$z=5$

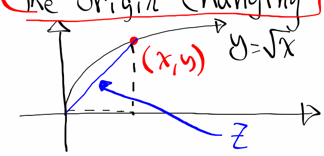
$$4(-4)+3(-3)=5\cdot\frac{dz}{dt}$$

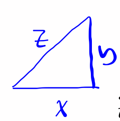
$$-25=5\cdot\frac{dz}{dt}\longrightarrow\frac{dz}{dt}=-5$$

ft/min.

An object is moving along the path given by $y=\sqrt{x}$

$\frac{dx}{dt}=4$ cm/min, $\frac{dy}{dt}=2$ cm/min. How fast is the distance between the object and the origin changing ($\frac{dz}{dt}$) when $x=4$.





$z^2=x^2+y^2$

$y=\sqrt{x}$

$y^2=x$

$z^2=x^2+x$

$z^2=4^2+4$

$z^2=20$

$z=\sqrt{20}$

$z^2=x^2+x$

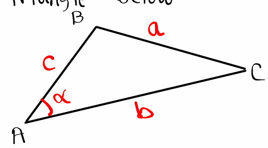
$2z\frac{dz}{dt}=(2x+1)\frac{dx}{dt}$

$2\cdot\sqrt{20}\frac{dz}{dt}=(2\cdot4+1)\cdot4$

$\frac{dz}{dt}=\frac{36}{2\sqrt{20}}=\frac{18}{\sqrt{20}}$

cm/min.

Consider the triangle below



$\overline{AB} = 8\,cm$

$\overline{AC} = 10\,cm$

$\frac{da}{dt} = 2$

Side BC is changing @ 2cm/min

How fast is angle A changing when $\measuredangle A = 45°$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\cancel{2}a \cdot \frac{da}{dt} = 0 - \cancel{2}bc \cdot (-\sin\alpha) \cdot \frac{d\alpha}{dt}$

$a\,\frac{da}{dt} = bc\,\sin\alpha \cdot \frac{d\alpha}{dt}$

$7.1 \cdot 2 = 10 \cdot 8 \cdot \sin 45° \cdot \frac{d\alpha}{dt} \Rightarrow \frac{d\alpha}{dt} = \boxed{}$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$= 10^2 + 8^2 - 2 \cdot 10 \cdot 8 \cdot \cos 45°$

$= 100 + 64 - 2 \cdot 80 \cdot \frac{\sqrt{2}}{2}$

$= 164 - 80\sqrt{2}$ $\quad a^2 = 50.863$ $\quad a \approx 7.1$

Exam 1: limits, Derivatives, Math.

You can start as early as 8:00 AM

You have until 10:30.